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Accurate Analysis Equations and Synthesis Technique for Unilateral Finlines

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Abstract —**Accurate analysis equations and synthesis techniques are presented for unilateral finlines, valid over a wide range of structural parameters and substrate dielectric constants ($1 \leqslant \epsilon_r \leqslant 3.75$). These expressions are usable for computing the cutoff wavelength to within ± 0.6 percent, the guided wavelength to within ± 2 percent, and the characteristic impedance (based on the power–voltage definition) to within ± 2 percent, of the spectral-domain method, over the normalized frequency range** $0.25 \leqslant b/\lambda \leqslant 0.6$.

I. Introduction

FINLINE IS AN ideal transmission line for millimeter-wave circuits because it avoids miniaturization and offers the potential for low-cost production through batch processing techniques [1], [2]. It is also easily compatible with semiconductor devices. It has wide bandwidth for single-mode operation, moderate attenuation, and low dispersion in the frequency range of interest. These properties have made it more popular than microstrip about 30 GHz.

Dispersion in finline has been accurately analyzed by Hofmann [3], Knorr and Shayda [4], Schmidt and Itoh [5], Beyer and Wolff [6], Sharma, Costache, and Hoefer [7], Shih and Hoefer [8], and Saad and Schunemann [9]. These analyses use the eigenmode analysis in space or the spectral-domain, finite-element method, or a two-dimensional transmission-line matrix. The network analytical method of electromagnetic fields, which is similar to the spectral-domain technique, was extended to the more general case of higher order modes by Hayashi, Farr, and Mittra [10]. Although the above-mentioned methods are highly accurate, they require considerable analytical effort and lead to complicated computer programming.

Besides the rigorous analyses above, the propagation constant in finlines has been approximated by various methods. Many authors have treated finlines as ridged waveguides [11], [12]. But the resulting expressions are of poor accuracy for the guided wavelength and the characteristic impedance. For an adequately accurate expression for the effective dielectric constant of finlines, one has to depend on experimental data [1] from expensive and time-consuming sample measurements. Therefore, in spite of all the advantages of a novel transmission line, the basic problem faced by the designers is the cumbersome design procedure.

Consequently, there remains a strong need for accurate closed-form expressions for the equivalent dielectric constant and characteristic impedance for finlines. Recently, Sharma and Hoefer [13] have presented purely empirical expressions for the cutoff wavelength of unilateral and bilateral finlines, which were developed by curve fitting to numerical results obtained by the spectral-domain technique [7]. Because of their purely empirical nature, these expressions are valid for a small range of finline geometries. For example, the equations are valid for $1/16 \leqslant d/b \leqslant 1/4$, $b/a = 0.5$, and $\epsilon_r = 2.22$ and 3.00 only (see Fig. 1(a)). Moreover, different equations are required for dielectric substrates of different permittivity values.

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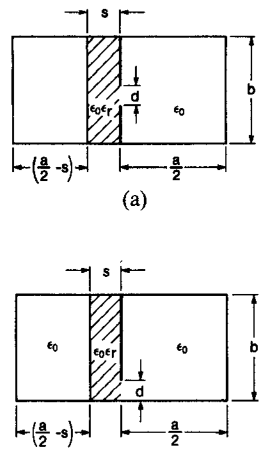

Fig. 1. (a) Unilateral finline. (b) Complementary unilateral finline.

Hence, the equations are of limited use and inappropriate for other dielectric substrates.

In the present work, accurate closed-form expressions are developed for the theoretical prediction of equivalent dielectric constants at cutoff and the cutoff wavelength of unilateral finlines. The expressions are accurate to within ± 0.6 percent. They have been derived using a stationary formula and curve fitting to numerical results of the spectral-domain method. The expressions are valid over the range (see Fig. 1(a))

$$0 < b/a \leqslant 1 \qquad 1/32 \leqslant d/b \leqslant 1.00$$
$$1/64 \leqslant s/a \leqslant 1/4 \qquad 1 \leqslant \epsilon_r \leqslant 3.75.$$

In addition, unlike Sharma and Hoefer's [13] expressions, the present expressions are less arbitrary and the constants appearing in them are independent of substrate dielectric constants for the most commonly used substrate materials.

II. Expressions for Guided Wavelength and Frequency-Dependent Characteristic Impedance

The guided wavelength in finline is defined as

$$\lambda_g = \lambda / \sqrt{\epsilon_e(f)} \tag{1}$$

where λ is the free-space wavelength and $\epsilon_e(f)$, the frequency-dependent effective dielectric constant of the finline, is given by [1]

$$\epsilon_e(f) = k_e - (\lambda/\lambda_{ca})^2 \tag{2}$$

where k_e is the equivalent dielectric constant at frequency f corresponding to wavelength λ, and λ_{ca} is the cutoff wavelength of a finned waveguide of identical dimensions and completely filled with air. It has been shown in [14] that for moderate $\epsilon_r (\epsilon_r \leqslant 2.5)$ and thin substrates one can make a first-order approximation by equating k_e to its value k_c at cutoff frequency. Otherwise, k_e must be considered frequency-dependent and has the general form [13]

$$k_e = k_c f(d/b, s/a, b/\lambda, \epsilon_r) \tag{3}$$

where

$$k_c = (\lambda_{cf}/\lambda_{ca})^2 \tag{4}$$

and λ_{cf} is the cutoff wavelength of the finline.

An empirical expression for the function $f(d/b, s/a, b/\lambda, \epsilon_r)$ in (3) is given in [13]. Having defined the effective dielectric constant $\epsilon_e(f)$, the characteristic impedance of the finline is defined as

$$Z_0 = Z_{0\infty} / \sqrt{\epsilon_e(f)} \tag{5}$$

where $Z_{0\infty}$, according to the ridged guide model [1], is the characteristic impedance at infinite frequency of the equivalent finned waveguide of identical dimensions.

III. Derivation of the Form of the Expression

The unilateral finline shown in Fig. 1(a) can be thought of as a combination of a finned waveguide and a waveguide loaded with a centered dielectric slab in the E-plane. The cutoff wavelength λ_{ca} of the finned waveguide can be determined accurately using the following equation [15]:

$$\frac{b}{\lambda_{ca}} = \frac{b}{2a}\left[1 + \frac{4}{\pi}\left(\frac{b}{a}\right)\left(1 + 0.2\sqrt{\frac{b}{a}}\right)\ln \operatorname{cosec}\left(\frac{\pi}{2}\frac{d}{b}\right)\right]^{-1/2}. \tag{6}$$

The cutoff wavelength λ_{cd} of the dielectric loaded waveguide can be obtained, for small s/a and ϵ_r, using the stationary formula [16], obtained from the variational technique, as

$$\frac{b}{\lambda_{cd}} = \frac{b}{2a}\left[1 + 0.5\left(\frac{2s}{a} + \frac{1}{\pi}\sin\left(\frac{2\pi s}{a}\right)\right)(\epsilon_r - 1)\right]^{-1/2}. \tag{7}$$

The derivation of (7) using the variational formula [16] assumes a sinusoidal field distribution in the transverse cross section of the guide. The field distribution has the form

$$\overline{E_y} = \bar{y}\sin\left(\frac{\pi x}{a}\right) \tag{8}$$

where $\bar{y}$ is the unit vector in the y-direction.

For a general field distribution in the transverse cross section of the waveguide, (7) can be written as

$$\frac{b}{\lambda_{cd}} = \frac{b}{2a}\left[1 + F\left(\frac{s}{a}\right)(\epsilon_r - 1)\right]^{-1/2} \tag{9}$$

where the function $F(s/a)$ depends upon the nature of the field distribution. Equation (9) has a stationary form. Therefore, it remains unchanged for small variations in ϵ_r, s/a, and frequency. For example, the inaccuracy of (7) is less than ± 0.8 percent for $s/a \leqslant 1/4$, which may be the thickest substrate used for finlines.

Now, consider the combination of the finned waveguide with the dielectric loaded waveguide. Due to the presence of the fins, the fields tend to concentrate in the vicinity of the fins, and the field distribution in the transverse cross section of the guide no longer remains a function of s/a only but also becomes a function of d/b.

Therefore, (9) assumes the form

$$\frac{b}{\lambda_{cd}}=\frac{b}{2a}\left[1+F\left(\frac{s}{a},\frac{d}{b}\right)(\epsilon_r-1)\right]^{-1/2}. \quad (10)$$

Once the function $F(s/a, d/b)$ is known, the cutoff wavelength λ_{cf} of the unilateral finline of Fig. 1(a) can be obtained using (2), (6), and (10) as

$$\frac{b}{\lambda_{cf}}=\frac{b}{2a}\left[\left\{1+\frac{4}{\pi}\left(\frac{b}{a}\right)\left(1+0.2\sqrt{\frac{b}{a}}\right)\ln\operatorname{cosec}\left(\frac{\pi}{2}\frac{d}{b}\right)\right\}\cdot\left\{1+F\left(\frac{s}{a},\frac{d}{b}\right)(\epsilon_r-1)\right\}\right]^{-1/2}. \quad (11)$$

IV. Derivation of the Function $F(s/a, d/b)$

Equation (11), when solved for $F(s/a, d/b)$, gives

$$F\left(\frac{s}{a},\frac{d}{b}\right)=\frac{\left[\left\{\left(\frac{\lambda_{cf}^2}{4a^2}\right)\Big/\left(1+\frac{4}{\pi}\left(\frac{b}{a}\right)\left(1+0.2\sqrt{\frac{b}{a}}\right)\ln\operatorname{cosec}\left(\frac{\pi}{2}\frac{d}{b}\right)\right)\right\}-1\right]}{(\epsilon_r-1)}. \quad (12)$$

Expressing

$$F\left(\frac{s}{a},\frac{d}{b}\right)=\left\{a_1\left(\frac{s}{a}\right)\ln\operatorname{cosec}\left(\frac{\pi}{2}\frac{d}{b}\right)+b_1\left(\frac{s}{a}\right)\right\}\frac{s}{a} \quad (13)$$

and using the accurately computed values of λ_{cf} from the spectral-domain technique [7] and (12), the function $F(s/a, d/b)$ can be computed for several combinations of s/a and d/b. The computed results are plotted in Fig. 2 ($F(s/a, d/b)(a/s)$ versus $\ln\operatorname{cosec}((\pi/2)(d/b))$ with s/a as a parameter). From the plots, we find that the curves are almost linear and the slopes and intercepts are functions of the s/a values. Slopes are larger for smaller s/a.

Using least-squares curve fitting gives the following set of equations for four different values of s/a for $\epsilon_r = 2.22$:

$$\left.\begin{aligned}a_1\left(\frac{s}{a}\right)&=0.1006616\\ b_1\left(\frac{s}{a}\right)&=1.6926529\end{aligned}\right\}\quad \text{for } \frac{s}{a}=\frac{1}{4} \quad (14)$$

$$\left.\begin{aligned}a_1\left(\frac{s}{a}\right)&=0.5339579\\ b_1\left(\frac{s}{a}\right)&=2.1643506\end{aligned}\right\}\quad \text{for } \frac{s}{a}=\frac{1}{8} \quad (15)$$

$$\left.\begin{aligned}a_1\left(\frac{s}{a}\right)&=1.353632\\ b_1\left(\frac{s}{a}\right)&=2.4213244\end{aligned}\right\}\quad \text{for } \frac{s}{a}=\frac{1}{16} \quad (16)$$

$$\left.\begin{aligned}a_1\left(\frac{s}{a}\right)&=2.5611088\\ b_1\left(\frac{s}{a}\right)&=2.3070609\end{aligned}\right\}\quad \text{for } \frac{s}{a}=\frac{1}{32}. \quad (17)$$

$a_1(s/a)$ and $b_1(s/a)$ are plotted as functions of $\ln(s/a)$ in Fig. 3. The two graphs can be represented by the equations

$$a_1\left(\frac{s}{a}\right)=0.4020974\left(\ln\left(\frac{a}{s}\right)\right)^2-0.7684487\ln\left(\frac{a}{s}\right)+0.3932021 \quad (18)$$

$$b_1\left(\frac{s}{a}\right)=2.42\sin\left(0.556\ln\left(\frac{a}{s}\right)\right). \quad (19)$$

From (10), the equivalent dielectric constant at cutoff is given by

$$k_c=1+\frac{s}{a}\left[a_1\left(\frac{s}{a}\right)\ln\operatorname{cosec}\left(\frac{\pi}{2}\frac{d}{b}\right)+b_1\left(\frac{s}{a}\right)\right](\epsilon_r-1) \quad (20)$$

which shows that k_c depends linearly on ϵ_r.

From (20), the dielectric filling factor can be written as

$$q_f=\frac{k_c-1}{\epsilon_r-1}=\frac{s}{a}\left[a_1\left(\frac{s}{a}\right)\ln\operatorname{cosec}\left(\frac{\pi}{2}\frac{d}{b}\right)+b_1\left(\frac{s}{a}\right)\right]. \quad (21)$$

Once k_c has been obtained, the frequency-dependent effective dielectric constant can be obtained using (2) and (3).

V. Characteristic Impedance

The definition of characteristic impedance for finlines is not unique. It depends upon the application. According to Meinel and Rembold [17], the characteristic impedance of a finline should be defined in terms of voltage and current in the finline, where voltage is defined as the line integral over the electric field between the fins taken along the shortest path on the substrate surface, and current is the total longitudinal current in the structure. This definition is useful for switching applications and particularly suitable for beam-lead devices. On the other hand, Meier [1] defines the characteristic impedance on a power–voltage basis and uses a ridged waveguide model for the finline. For his model, the term $Z_{0\infty}$ in (5) is frequency-independent. But, in practice, $Z_{0\infty}$ rises slowly with frequency as the crowding of the electric field causes the fin gap voltage to rise. This has been observed by computation of Z_0 using the network analytical technique [10]. Keeping the above facts in mind, the following expression for Z_0 has been derived by curve fitting to the spectral-domain results:

$$Z_0=\frac{240\pi^2(px+q)(b/a)}{(0.385x+1.762)^2\sqrt{\epsilon_e(f)}} \quad (22)$$

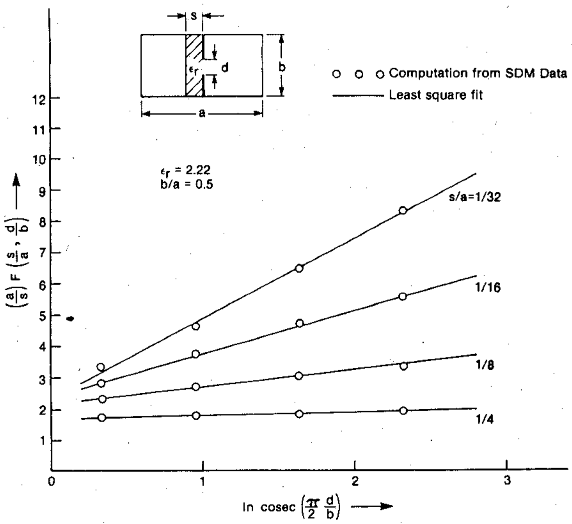

Fig. 2. Least-squares curve fitting to SDM data.

where

$$p=-0.763\left(\frac{b}{\lambda}\right)^2+0.58\left(\frac{b}{\lambda}\right)+0.0775\left[\ln\left(\frac{a}{s}\right)\right]^2 -0.668\left[\ln\left(\frac{a}{s}\right)\right]+1.262 \quad (23)$$

$$q=0.372\left(\frac{b}{\lambda}\right)+0.914, \qquad \text{for } d/b>0.3 \quad (24)$$

and

$$p=0.17\left(\frac{b}{\lambda}\right)+0.0098 \quad (25)$$

$$q=0.138\left(\frac{b}{\lambda}\right)+0.873, \qquad \text{for } d/b\leqslant 0.3 \quad (26)$$

and

$$x=\ln \operatorname{cosec}\left(\frac{\pi}{2}\frac{d}{b}\right). \quad (27)$$

Equation (22) is accurate within ± 2 percent for $s/a \leqslant 1/20$ and within ± 3 percent for $s/a > 1/20$, while Meier's ridged waveguide model [1] has inaccuracies of the order of ± 9 percent [4].

VI. Synthesis

To fabricate the appropriate finline structure, one requires the normalized fin-gap d/b for a given set of s/a, b/a, and ϵ_r. This can be obtained from (22) in the following way.

For low dielectric constant and thin substrate, the frequency-dependent effective dielectric constant can be written as

$$\epsilon_e(f)=k_c-\left(\frac{\lambda}{\lambda_{ca}}\right)^2. \quad (28)$$

Using (20) and noting that

$$\frac{\frac{s}{a}a_1\left(\frac{s}{a}\right)(\epsilon_r-1)x-\left(\frac{\lambda}{\lambda_{ca}}\right)^2}{1+\frac{s}{a}b_1\left(\frac{s}{a}\right)(\epsilon_r-1)} \ll 1$$

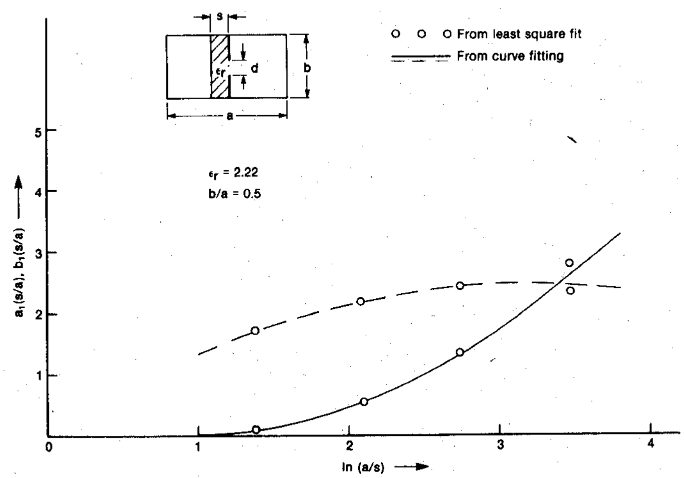

Fig. 3. Curves representing $a_1(s/a)$ and $b_1(s/a)$ a function of $\ln(a/s)$(——$a_1(s/a)$- - - -$b_1(s/a)$).

for $0 < b/a \leqslant 1$, $1/32 \leqslant d/b \leqslant 1$, $s/a \leqslant 1/20$, $1 \leqslant \epsilon_r \leqslant 3.75$, and $0.1 \leqslant b/\lambda \leqslant 0.6$, one can write

$$\sqrt{\epsilon_e(f)}=\frac{Gx^2+Hx+I}{Fx+E} \quad (29)$$

where

$$E=8\left[1+\frac{s}{a}b_1\left(\frac{s}{a}\right)(\epsilon_r-1)\right]^{1/2} \quad (30)$$

$$F=\left(\frac{4}{\pi}\right)\left(\frac{b}{a}\right)\left(1+0.2\sqrt{\frac{b}{a}}\right)E \quad (31)$$

$$G=0.5\left(\frac{s}{a}\right)a_1\left(\frac{s}{a}\right)(\epsilon_r-1)F\Big/\left[1+\frac{s}{a}b_1\left(\frac{s}{a}\right)(\epsilon_r-1)\right]^{1/2} \quad (32)$$

$$H=E(F/8+G/F) \quad (33)$$

$$I=E^2/8-\left(\frac{b}{a}\right)^2(\lambda/b)^2. \quad (34)$$

Combining (22) and (29) gives

$$Z_0=\frac{240\pi^2(px+q)(Fx+E)(b/a)}{(0.385x+1.7621)^2(Gx^2+Hx+I)} \quad (35)$$

for a given set of Z_0, b/a, s/a, ϵ_r, and b/λ, (35) reduces to a quartic equation in x of the form

$$x^4+C_3x^3+C_2x^2+C_1x+C_0=0 \quad (36)$$

where

$$C_3=H/G+9.156 \quad (37)$$

$$C_2=I/G+9.156(H/G)+20.95-6.748\frac{pF}{G\overline{Z}} \quad (38)$$

$$C_1=9.156(I/G)+20.95(H/G)-6.748(pE+qF)/G\overline{Z} \quad (39)$$

$$C_0=20.95(I/G)-6.748(qE/G\overline{Z}) \quad (40)$$

$$\overline{Z}=Z_0/\{(240\pi^2)(b/a)\}. \quad (41)$$

Equation (36) can be solved easily using a suitable iterative

TABLE I

Normalized substrate Thickness s/a	Normalized Gap Width d/b	Cutoff Frequency b/λ_{cf} of the dominant mode $\epsilon_r = 3.00$ SDT*	SH**	PM***
1/4	1/2	0.16269	-	0.16363
	1/4	0.13908	0.13840	0.13941
	1/8	0.12146	0.12233	0.12169
	1/16	0.10874	0.10814	0.10883
1/8	1/2	0.17706	-	0.17731
	1/4	0.14756	0.14673	0.14853
	1/8	0.12684	0.12800	0.12741
	1/16	0.11244	0.11167	0.11208
1/16	1/2	0.19114	-	0.19147
	1/4	0.15799	0.15640	0.15853
	1/8	0.13410	0.13502	0.13448
	1/16	0.11755	0.11657	0.11711
1/32	1/2	0.20275	-	0.20396
	1/4	0.16881	0.16766	0.16844
	1/8	0.14285	0.14364	0.14253
	1/16	0.12409	0.12306	0.12386

*Spectral domain technique [7]
**Empirical equation by Sharma and Hoefer [13]
***Present method

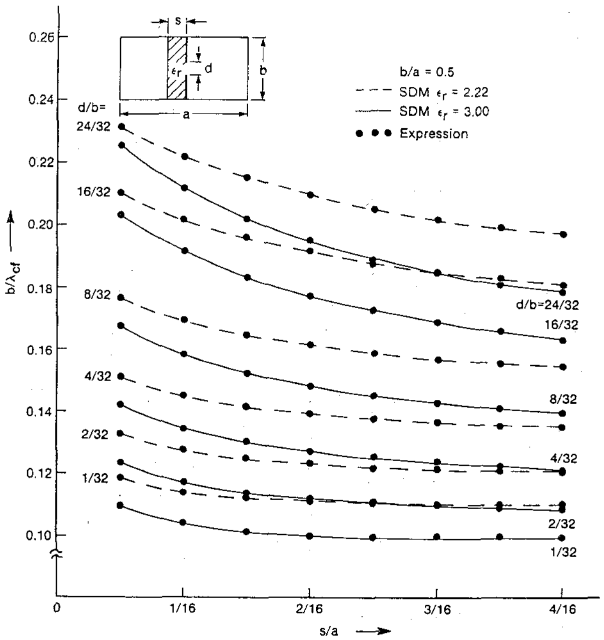

Fig. 4. Normalized cutoff frequency (b/λ_{cf}) as a function of normalized fin-gap $(d/b)\epsilon_r = 2.22$ and 3.00, $b/a = 0.5$.

technique to determine x, and hence d/b, since

$$d/b = \left(\frac{2}{\pi}\right)\sin^{-1}[\exp(-x)]. \tag{42}$$

VII. Computed Results

The results computed using (11), (18), (19), and (20) are shown in Table I and compared with those obtained using the spectral-domain technique [7] and the empirical expressions of Sharma and Hoefer [13]. The results show excellent agreement. Since the expressions were derived using the spectral-domain technique data for $\epsilon_r = 2.22$, the excellent agreement of the expression when $\epsilon_r = 3.00$ confirms the stationary nature of (20). Hence, (20) is a general equation for low ϵ_r values. The expressions have been found valid for $\epsilon_r = 3.75$ within ± 1 percent.

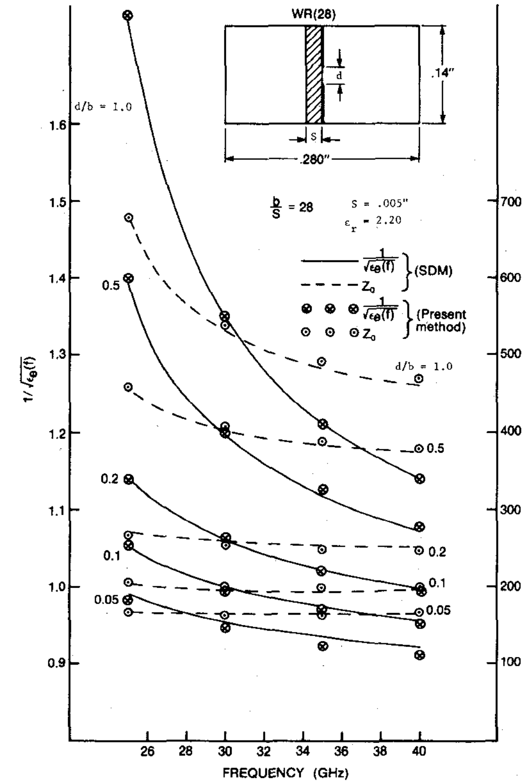

Fig. 5. Normalized phase velocity $1/\sqrt{\epsilon_e(f)}$ versus frequency and characteristic impedance Z_0 versus frequency in WR(28) waveguide. $s/a = 0.01786$.

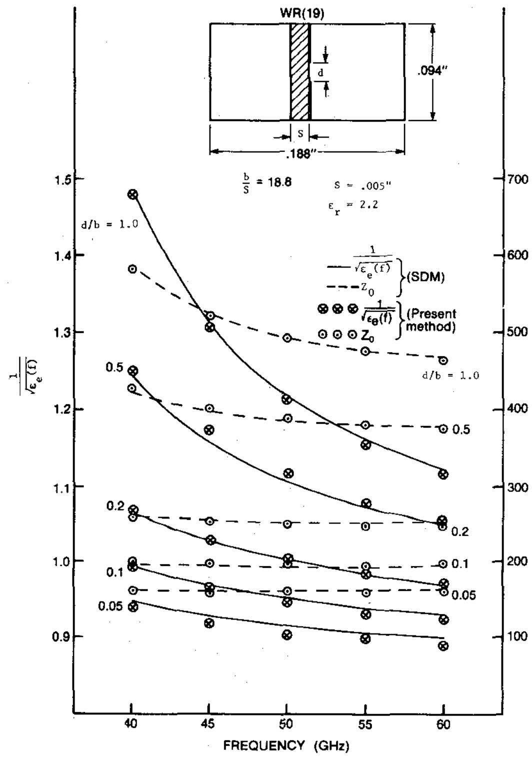

Fig. 6. Normalized phase velocity $1/\sqrt{\epsilon_e(f)}$ versus frequency and characteristic impedance versus frequency in WR(19) waveguide. $s/a = 0.0265957$.

TABLE II

d/b	Frequency (GHz)	Z_0 (D) (Ohm)	Z_0 (S) (Ohm)	$\frac{1}{\sqrt{\epsilon_e(f)}}$ (D)	$\frac{1}{\sqrt{\epsilon_e(f)}}$ (S)
	40	258.754	256.689	1.0773	1.0743
	45	252.733	248.420	1.0337	1.0292
0.2	50	249.338	244.012	1.00495	0.99968
	55	247.180	241.716	0.98462	0.9788
	60	245.556	240.695	0.96949	0.96321
	65	244.078	240.499	0.95777	0.95105
	40	429.924	433.563	1.2768	1.2986
	45	403.359	405.294	1.1797	1.1864
0.5	50	389.174	391.762	1.1222	1.1239
	55	381.068	381.429	1.0843	1.0840
	60	376.327	375.360	1.0578	1.0565
	65	373.585	371.724	1.03822	1.0363

D ≜ Desired
S ≜ Synthesized

Fig. 4 compares the normalized dominant-mode cutoff wavelength (b/λ_{cf}) in unilateral finlines, computed by the present method, with those obtained by the spectral-domain technique [7] for $\epsilon_r = 2.22$ and 3.00. Figs. 5 and 6 show the variations of normalized guided wavelength and characteristic impedance with frequency and compare with the results of Knorr and Shayda [4]. The agreement is within ± 2 percent. This inspires confidence in the above expressions.

The results of synthesis are shown in Table II for 127-μm RT Duroid substrate in WR(19) waveguide. Z_0 values were calculated, over the useful frequency band, using (22). The computed values of Z_0 were then used in (36)–(41) to compute x. The computed values of x were then used to obtain the synthesized Z_0 and $\epsilon_e(f)$ values from the analysis equations. It is found that the synthesized Z_0 are within ± 3 percent and guided wavelength is within ± 2 percent of design specifications. Therefore, the synthesis technique may be used to obtain an initial design, after which the more accurate analysis equation could be employed to correct the design dimensions, if required.

VIII. Application to Complementary Unilateral Finline

The closed-form expressions derived above for analysis of the unilateral finline can be easily applied to the complementary unilateral finline, shown in Fig. 1(b), with the following interpretations: b is twice the height of the housing of the complementary finline, and d is twice the gap between the tip of the fin and the bottom wall of the housing. With this, (20) remains unchanged for the complementary unilateral finline.

The interpretation remains the same for the characteristic impedance. But, the characteristic impedance of the complementary unilateral finline is half that of the unilateral finline.

IX. Conclusions

In the preceding sections, accurate closed-form expressions are developed for the cutoff wavelength and the equivalent dielectric constant at cutoff of unilateral finlines. The derivation starts with the assumption of a stationary form for the equivalent dielectric constant at cutoff. The stationary function is subsequently determined by least-squares curve fitting to accurate numerical data, obtained by the spectral-domain technique. The expressions are accurate within ± 0.6 percent over the complete practical range of interest. They can be used for quick and easy evaluation of the dispersion characteristic by hand or calculator within ± 2 percent, for $\epsilon_r \leqslant 2.50$ and $b/\lambda \leqslant 0.6$. For higher ϵ_r, a frequency correction of the equivalent dielectric constant is required.

An expression is developed for the characteristic impedance, based on the power–voltage definition, by curve fitting to results obtained by the spectral-domain technique. The expression is accurate to within ± 2 percent for all practical purposes.

The first-order synthesis technique gives the phase velocity within ± 2 percent and the characteristic impedance within ± 3 percent of originally specified values. They can be further corrected using the more accurate analysis formulas. The present analysis and synthesis equations will be useful in computer-aided design and optimization of unilateral finline circuits [18].

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General Stability Analysis of Periodic Steady-State Regimes in Nonlinear Microwave Circuits

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***Abstract* —The problem of analyzing the stability of periodic equilibrium regimes in nonlinear microwave circuits is tackled by a general-purpose computer-aided approach. By means of a perturbation technique, the search for instabilities is reduced to a generalized eigenvalue equation expressed in matrix form, and is then carried out by Nyquist's analysis. The use of a vector processor allows the computer time requirements to be kept well within reasonable limits, even in the case of large-size problems. In perspective, this could open the way to the complementation of existing nonlinear CAD packages by an on-line facility for automatic stability analysis.**

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I. Introduction

THIS PAPER is devoted to introducing a new numerical technique for analyzing the stability of periodic steady-state regimes in nonlinear microwave circuits. This problem is a very difficult one, and has been tackled in the literature by a variety of approximations and limiting assumptions (e.g., [1]–[6]). From time to time, the analysis has been restricted to specific kinds of nonlinear devices, and/or the representation of the perturbed regime has been severely simplified, either by reducing the number of spectral lines to be accounted for, or by resorting to the concept of slowly changing perturbation.

On the other hand, the emphasis here is on generality. At least in principle, our all-computer approach should be